Gear Reducer Design

This document is for the design of a reverted gear train for speed reduction. Gears 3 and 4 are compounded (locked in common motion).

The gear box will use spur gears and to make the design simpler and cost lower 2 pairs of the same gear will be used.

1.0 Design Specifications

Power to be Delivered	20 kW
Input Speed	1800 rpm
Output Speed Range	85 – 95 rpm
Max. Gearbox Size	600 x 400 x 400 mm
Gear and Bearing Life	> 15 000 hrs

The gearbox will also have infinite shaft life and it will be subjected mostly to low shock and occasionally moderate shock.

2.0 Initial Diagram

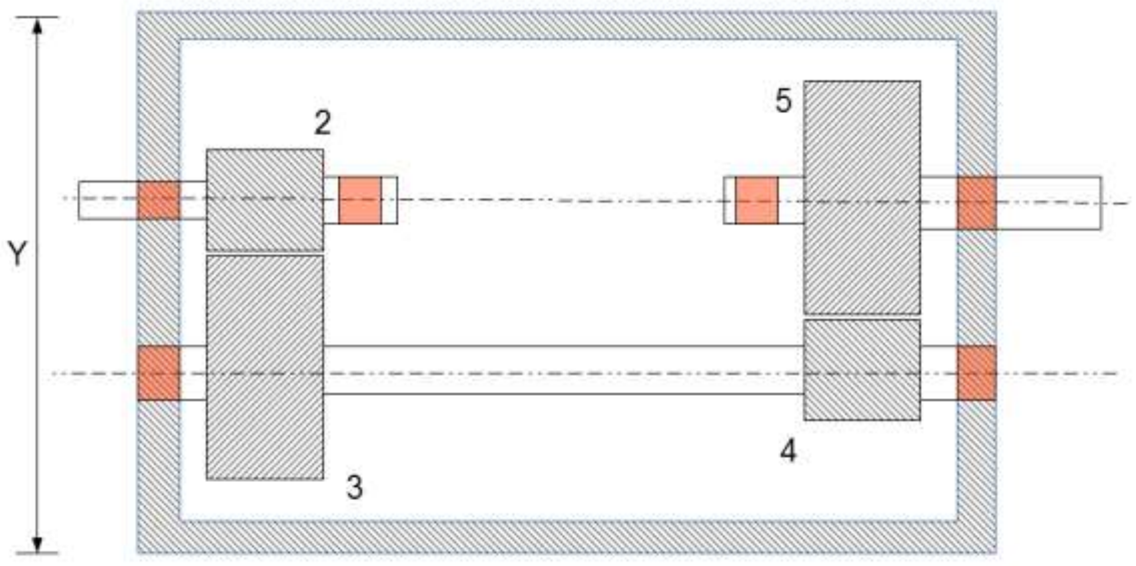

Figure 1 – Overview diagram of gear box. Bearing locations are shaded in orange. Gears are numbered 2 to 5.

2.0 Power and Torque

Assuming that the gearbox is 100% efficient and that power in is equal to power out then the following can be written.

$$H = T_i\,\omega_i = T_o\omega_o$$

The angular speed ratio is the same as the inverse torque ratio. This is also the ratio between the product of driving gear teeth and the product of driven gear teeth.

$$e = \frac{T_i}{T_o} = \frac{\omega_o}{\omega_i}$$

$$e = \frac{product\ of\ driving\ teeth}{product\ of\ driven\ teeth}$$

The range of acceptable output speed is between 85 and 95 rpm. Take the mean value of this range, 90 rpm, and use it as the output speed.

$$e = \frac{90}{1800} = \frac{1}{20}$$

For the smallest size gearbox let both stages be the same reduction. This also satisfies the 'in-line' condition for the parallel input and output shafts in the reverted gear train.

$$e = \frac{N_2 x N_4}{N_3 x\ N_5}$$

$$\frac{N_2}{N_3} = \frac{N_4}{N_5} = \sqrt{e} = \frac{1}{\sqrt{20}}$$

To find the minimum number of teeth on the pinion (smallest gear) that can be there without interference an equation is used.

The module of the gear is the pitch diameter divided by the number of teeth. A common module (m) for the size of these gears is 4.

$$N_p = \frac{2k}{(1+2m)\sin^2\theta}\left(m + \sqrt{(m^2 + (1+2m)\sin^2\theta}\right.$$

Where:

k = 1 for full depth teeth.

θ= 25˚ = as it is the pressure angle of the tooth. This is the angle between the line of force that the teeth exert on each other and the tangent line of the pitch circle. In general it has the values of either 14.5, 20 and 25 degrees. I have chosen 25 degrees as interference can be reduced by using larger angles and smaller base circles. However greater frictional forces and there are larger bearing loads.

$$N_p = 10.19 = 11\ teeth.$$

Always round up for teeth numbers.

Therefore:

$$N_2 = N_4 = 11\ teeth$$

So:

$$N_5 = N_3 = \frac{\sqrt{20}}{1}\ x\ 11 = 49.19\ teeth = 49\ teeth$$

Use these teeth numbers to check that the output is still in the speed range:

$$e = \frac{N_2 x N_4}{N_3 x N_5}$$

$$\omega_o = e\ x\ \omega_i$$

$$\omega_o = \frac{N_2 x N_4}{N_3 x N_5}\ x\ 1800\ = 90.71\ rpm$$

This is only 0.71 rpm from the mean of the range so the teeth numbers are good values. With this information we can also find the angular speed of gears 3 and 4, it is the same as they are locked to the same shaft:

$$\omega_3 = \omega_4 = \frac{11}{49}\ x\ 1800 = 404.1\ rpm$$

So:

$$N_2 = N_4 = 11\ teeth$$
$$N_3 = N_5 = 49\ teeth$$
$$\omega_5 = 90.71\ rpm$$
$$\omega_3\ = \omega_4 = 404.1\ rpm$$

Since power in is equal to power out and power is the product of the torque times the angular speed (in radians per second)! The torque on all of the gears can be calculated:

$$H = T_i\,\omega_i = T_o\omega_o$$

$$T_2 = \frac{H}{\omega_2} = \frac{20\,kW}{1800\,rpm} = \frac{20{,}000}{60\,rad\,s^{-1}}$$

$$T_2 = 106.10\,Nm$$

$$T_3 = T_4 = T_2\left(\frac{\omega_2}{\omega_3}\right) = 106.10\,\left(\frac{1800}{404.1}\right) = 472.62\,Nm$$

$$T_5 = T_2\left(\frac{\omega_2}{\omega_5}\right) = 106.1\left(\frac{1800}{90.71}\right) = 2{,}105.39\,Nm$$

3.0 Size of Gearbox and Diametral Pitch

From figure 1 the height of the gearbox needs to be used to figure out other aspects of the gears. The diametral pitch (P) or teeth per diameter length needs to be found. From the specifications Y has to be equal to 600mm. Y needs to be put in terms of the gear diameters, clearances and wall thickness.

$$Y = d_3 + \frac{d_2}{2} + \frac{d_5}{2} + \frac{2}{P} + clearances + wall\ thicknesses$$

* As the diameter of the gears is terms of the pitch circle the addendums of gear 3 and 2 need to be taken into account. This is the 2/P term as for a pressure angle of 25˚ addendum=1/P.

As:

$$P = \frac{N}{d}$$

$$d = \frac{N}{P}$$

So:

$$Y = \frac{N_3}{P} + \frac{N_2}{2P} + \frac{N_5}{2P} + \frac{2}{P} + clearances + wall\ thicknesses$$

Solving for P:

$$P = \frac{\left(N_3 + \frac{N_2}{2} + \frac{N_5}{2} + 2\right)}{Y - clearances - wall\ thicknesses}$$

As stated max size is 600mm and allowing 50mm for clearances and wall thicknesses.

$$P_{min} = \frac{49 + \frac{11}{2} + \frac{49}{2} + 2}{(600 - 50)} = 147.3x10^{-3} teeth/\ mm$$

$$P_{min} = \ 3.68\ teeth/\ inch$$

So:

$$d_2 = d_4 = \frac{N_2}{P} = \frac{11}{147.3x10^{-3}} = 74.68\ mm$$

$$d_3 = d_5 = \frac{N_3}{P} = \frac{49}{147.3x10^{-3}} = 332.65mm$$

4.1 Pitch Line Velocity and Transmitted Loads

Now that the pitch diameters and the angular speeds of all the gears is found out the pitch line velocities (V) and the transmitted loads (W) can be found for later use.

Pitch Line Velocity

The pitch line velocities for gear 2 and 3 are the same and the pitch line velocities for gear 4 and 5 are the same.

$$V = \pi dn$$

Where:

n=gear speed in **rev/s**

$$V_{2\&3} = \pi d_2 n_2 = \pi(74.68)\left(\frac{1800}{60}\right) = 7038mm/s$$

$$V_{4\&5}\ = \pi d_3 n_3 = \pi(332.65)\left(\frac{90.71}{60}\right) = 1580\ mm/s$$

Transmitted Loads

Assuming that the gear box is 100% efficient in transferring loads the equation for transmitted loads is:

$$W_t = \frac{60{,}000H}{\pi dn}$$

Where:

Power (H) is in kW

W_t is in kN

n is in **rev/min** . Pitch line velocity values can be used in place of denominator if it is multiplied by 60.

Once again the value for transmitted loads is the same for gears 2 and 3. The valued for transmitted loads for gears 4 and 5 is the same.

$$w_{t(2\&3)} = \frac{60{,}000(20)}{7038x60} = 2.84kN$$

$$W_{t(4\&5)} = \frac{60{,}000(20)}{1580x60} = 12.66kN$$

5.0 Wear and Bending of Gear 4

Start with gear 4 as it is the smallest and transmits the largest load. To find the amount of wear on gear 4 the allowable contact stress (S_c) needs to be figured out. After this is done a material can be assigned to the gear so that it does not fail due to wear over its specified life.

5.1 Wear

Need to find allowable contact stress (S_c)

$$\sigma_c = \frac{S_c}{S_H}\frac{Z_n Z_w}{Y_\theta Y_z}$$

Therefore:

$$S_c = \frac{\sigma_c S_H Y_\theta Y_Z}{Z_N Z_W}$$

Z_N =Stress life cycle factor

S_H = Safety Factor

Y_Z = Reliability Factor

Y_θ= Temperature factor

Z_W = Hardness ratio for pitting resistance

σ_c = Contact stress

Find Contact Stress (AGMA Relationship)

$$\sigma_c = Z_E \sqrt{W_t K_o K_v K_s \frac{K_H}{d_w b} \frac{Z_R}{Z_1}}$$

Already know W_t

Z_1 or I = geometry factor of contact stress or pitting resistance

K_V = Dynamic factor

K_O = overload factor

K_S = Size factor

K_H = Load distribution factor

Z_R = Surface condition factor

d_w = Pitch diameter of pinion (mm)

b = Width of face at narrowest member

Z_E = Elastic coefficient (depends on material of gears)

5.1.1 Find Geometry Factor of contact stress

As gear 4 is an external gear:

$$I = \frac{cos\theta sin\theta}{2m_N} \frac{m_G}{m_G + 1}$$

m_N = 1 for spur gears

m_G =speed ratio = 49/11 =4.45

θ is the pressure angle of 25˚

$$I = \frac{cos25sin25}{2(1)} \frac{4.45}{4.45 + 1} = 0.156$$

5.1.2 Find Dynamic Factor

Need to find K_V in m/s

$$K_V = \left(\frac{A + \sqrt{200V}}{A}\right)^B$$

Where:

We already know V the pitch line velocity for gear 4 = 1.58m/s

$$A = 50 + 56(1 - B)$$
$$B = 0.25(12 - Q_V)^{\frac{2}{3}}$$

Q_v is the transmission accuracy level and ranges in value 6 to 12. Smaller numbers represent higher accuracy. Let Q_v =7.

$$B = 0.25(12 - 7)^{\frac{2}{3}} = 0.73$$
$$A = 50 + 56(1 - 0.73) = 65.06$$

So:

$$K_V = \left(\frac{65.06 + \sqrt{200x1.58}}{65.06}\right)^{0.73} = 1.193$$

5.1.3 Overload Factor

The overload factor was taken from a table and as in the specifications there is uniform input power and the gear reducer will be subjected to occasional moderate shock the value of K_o =1.25

5.1.4 Tooth Size Factor (K_s)

This will be set as 1 as it is recommended by AGMA.

5.1.5 Load Distribution Factor (K_H)

Inches will be used in this section as it is a factor this will not influence the end contact stress.

$$K_H = C_{mf} = 1 + C_{mc}\left(C_{pf}C_{pm} + C_{ma}C_e\right)$$

C_{mc} =1 for uncrowned teeth

C_{pm}=1 for straddled mounted pinion where pinion offset from centre span divided by bearing span is less than 0.175

C_e = 1

$$C_{ma} = A + BF + CF^2$$

A,B and C are taken from table where the values are for commercial enclosed unit.

Need to find face width, F. This is also b.

Face width is normally 10/P

Since:

$$P = 0.1473\ teeth/mm\ P = 3.68\ teeth\ /inch$$

$$F = \frac{10}{3.68} = 2.72\ inches = \ 67.88\ mm$$

So:

$$C_{ma} = \ 0.127 + 0.0158(2.72) + (-0.93x10^{-4})(2.72)^2$$
$$C_{ma} = 0.1699$$

When 1<F<17 inches

$$C_{pf} = \frac{F}{10d} - 0.0375 + 0.0125F$$
$$C_{pf} = \frac{2.72}{29.9} - 0.0375 + (0.0125)(2.72)$$
$$C_{pf} = 0.0875$$

So:

$$K_H = 1 + 1\big((0.0875x1) + (0.1699\ x1)\big)$$
$$K_H = 1.2574$$

5.1.6 Surface Condition (Z_R)

It is recommended that $Z_R = 1$.

5.1.7 Elastic Coefficient (Z_e)

The elastic coefficient is taken from a table of gear and pinion materials. Both gear 4 and 5 will be made of steel which has an elastic coefficient of 191MPa

Since we know all of the unknowns in the equation

$$\sigma_c = Z_E \sqrt{W_t K_o K_v K_s \frac{K_H}{d_w b} \frac{Z_R}{Z_1}}$$

$$\sigma_c = 191 \sqrt{(12660)(1.25)(1.193)(1) \left(\frac{1.2574}{74.83x67.88}\right) \left(\frac{1}{0.156}\right)}$$
$$\sigma_c = 1045.42\ MPa$$

5.1.8 Life Factor

Need to calculate the number of rotations the gear will do over its estimated life cycle

$$L = (15{,}000hrs)\left(\frac{60mins}{hr}\right)(404.1\ rpm) = 3.637x10^{8}\ rev$$

Using a graph this gives a stress cycle factor (Z_n) of 0.9.

If:

$Y_\theta = Y_Z = Z_W = 1$

And the safety factor is 1.2

$$S_c = \frac{\sigma_c S_H}{Z_N}$$

$$S_C = \frac{1045.42\ x\ 1.2}{0.9} = 1393.9MPa$$

From a table of allowable contact stresses of materials Grade 3 Nitralloy N would be suitable. It has a allowable contact stress of 1413.4 MPa.

To find the actual factor of safety:

$$S_H = \frac{(S_C\ Z_N)}{\sigma_c}$$

$$S_H = \frac{(1413.4)(0.9)}{1045.42} = 1.217$$

5.2 Bending

The equation for allowable bending stress is:

$$\sigma = W_t K_o K_V K_s \frac{P}{b}\frac{K_H}{J}$$

Need to find geometry factor J. Everything else will be the same as in the wear section.

$$J = \frac{Y}{K_f m_n}$$

Where m_n is 1 for spur gears.

Where K_f is the stress correction value and is normally around 1.1 for these sizes of gears.

Y is given from tables and has a value of 0.245 for 11 teeth.

$$J = \frac{0.245}{1.1} = 0.22$$

So:

$$\sigma = (12660)(1.25)(1.195)(1)\left(\frac{0.147}{67.88}\right)\left(\frac{1.2574}{0.22}\right)$$
$$\sigma = 233.99\ MPa$$

Y_N = Stress cycle factor and is taken from a graph and has the value of 0.9

The Grade 3 Nitralloy N has a allowable bending stress of 275MPa .

The actual factor of safety:

$$n = \frac{\sigma_{alloy} Y_N}{\sigma}$$

$$n = \frac{(275x\ 0.9)}{233.99} = 1.06$$