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DESIGN OF A WING SPAR

Design Engineering – Engineering Simulation (Mechanics) – Year 2

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Question 1

Question – If spar is in the form of an I-section beam with its cross section shown below, calculate all of its section properties of the centroid, I_{xx} , I_{yy} , I_{xy} , I_{max} and I_{min} and the direction of the principal axis for the section relative to the X axis.

1.1 Calculating I_{xx} :

(Looking at Figure 2 and Figure 1 the B and D values can be calculated.)

$$I_{xx} = \frac{BD^3}{12} - 2\left(\frac{bd^3}{12}\right)$$

$$I_{xx} = \frac{80 \times 120^3}{12} - 2\left(\frac{38 \times 110^3}{12}\right)$$

$$I_{xx} = 11.52 \times 10^6 - 8.430 \times 10^6$$

$$I_{xx} = 3.0903 \times 10^6 \text{ mm}^4$$

$$I_{xx} = 3.0903 \times 10^{-6} \text{ m}^4$$

1.2 Calculating I_{yy} :

(Looking at Figure 3 and Figure 1 the B and D values can be calculated)

$$I_{yy} = 2\left(\frac{BD^3}{12}\right) + \frac{bd^3}{12}$$

$$I_{yy} = 2\left(\frac{5 \times 80^3}{12}\right) + \frac{110 \times 4^3}{12}$$

$$I_{yy} = \frac{1280000}{3} + \frac{1760}{3}$$

$$I_{yy} = 427.25 \times 10^3 \text{ mm}^4$$

$$I_{yy} = 427.25 \times 10^{-9} \text{ m}^4$$

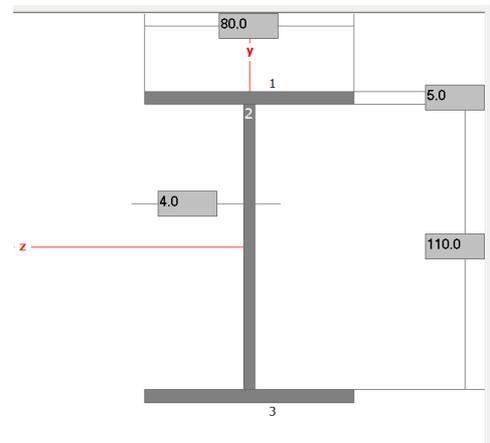


Figure 1- I-section beam dimensions

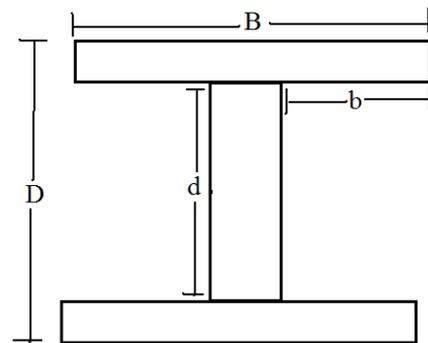


Figure 2- I section denotations for I_{xx}

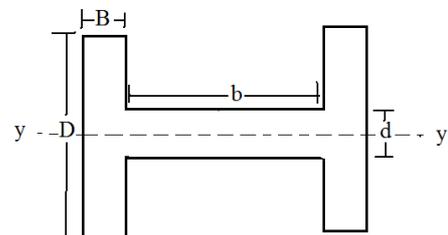


Figure 3- I section denotations for I_{yy}

1.3 Calculating I_{xy}:

I_{xy} = 0 . As there are two axes of symmetry about horizontal and vertical lines where the origin is located at the centroid (see section 1.4.) If there are any axes of symmetry then I_{xy} is equal to 0 because of:

$$\int xy dA = \int x dx \int y dy$$

As there is no change in dx and dy between the centroidal axes and the principal axes the so it must equal 0.

1.4 Calculating the Centroid of the Cross-Section

Areas (Figure 4) - 1&3: 80 x 5 = 400mm²
 2: 4 x 110 = 440 mm²
 Sum of areas = 400+440+400 = **1,240mm²**

1.4.1 Horizontal Component:

$$\bar{x} = (\Sigma(x A))/(\Sigma(A))$$

$$\bar{x} = \frac{(40 \times 400) + (40 \times 440) + (40 \times 400)}{400 + 440 + 400}$$

$$\bar{x} = \frac{16,000 + 17,600 + 16,000}{1,240}$$

$$\bar{x} = \mathbf{40 \text{ mm}}$$

1.4.2 Vertical Component:

$$\bar{y} = \frac{\Sigma(y A)}{\Sigma A}$$

$$\bar{y} = \frac{(2.5 \times 400) + (60 \times 440) + (117.5 \times 400)}{400 + 440 + 400}$$

$$\bar{y} = \frac{1,000 + 26,400 + 47,000}{1,240}$$

$$\bar{y} = \mathbf{60 \text{ mm}}$$

So the Cartesian Co-ordinates for the centre of the section is (40, 60). From the bottom left corner of the section shown as a red dot in Figure 4.

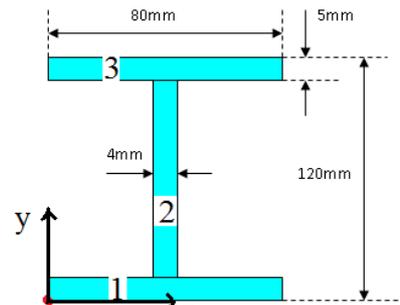


Figure 4 –CSA broken down for calculations

Z Axis Properties			
From bottom to centroid	y (bot)	60.0000	mm
From centroid to top	y (top)	60.0000	mm
Area of shape	A	1,240.0000	mm ²
Moment of Inertia	Iz	3.0903E+06	mm ⁴
Section Modulus	Sz	51,505.5556	mm ³
Section Modulus (bottom)	S (bot)	51,505.5556	mm ³
Section Modulus (top)	S (top)	51,505.5556	mm ³
Radius of Gyration	rz	49.9220	mm
Plastic Modulus	Zz	58,100.0000	mm ³
Shape Factor		1.1280	
From bottom to plastic n.a.	yp (bot)	60.0000	mm
From plastic n.a. to top	yp (top)	60.0000	mm
Polar Moment of Inertia	J	3.5176E+06	mm ⁴
Product of Inertia	Iyz	0.0000	mm ⁴
Maximum Moment of Inertia	I _{max}	3.0903E+06	mm ⁴
Minimum Moment of Inertia	I _{min}	427,253.3333	mm ⁴
Angle from z axis to I _{max} axis	B	0.0000	degrees
		Counterclockwise	

Question 2

If the spar is modelled as a uniform horizontal cantilever with various loads (lift, weight, Engine weight, undercarriage effect, etc.) transferred to it as shown below:

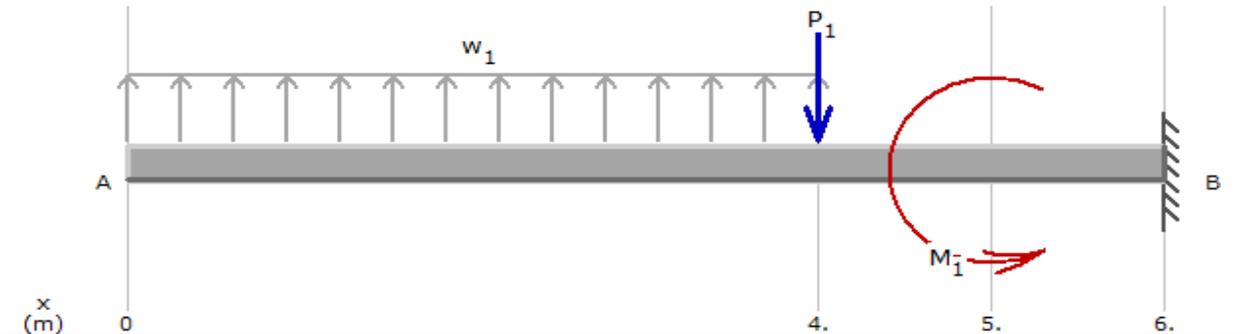


Diagram 1- Loads on the Beam

$$W_1 = 2\text{kN/m}$$

$$M_1 = 10\text{kNm}$$

$$P_1 = 10\text{kN}$$

Draw the shear force and bending moment diagrams for this beam. Assume material is Al alloy with $E=70\text{GPa}$ & $\nu=0.3$

2.1 Hand Drawn

2.1.1 Shear Force Diagram

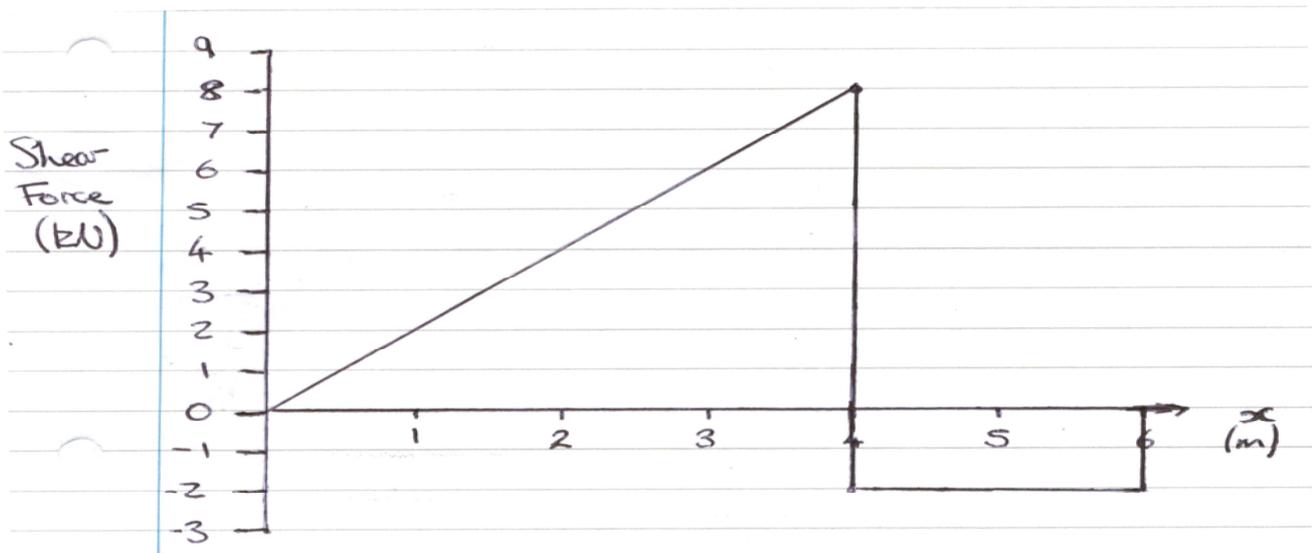


Diagram 2- Drawn shear force diagram

- i. The wing spar is modelled as a cantilever beam that is free at the end, the end that is free is at co-ordinates (0, 0) in diagram 2. The shear force here starts at 0 as there are no reaction forces here and there are no loads acting at this point.
- ii. In diagram 1 there is a UDL of 2kN/m acting over a distance of 4m this means that at the 4m point there is a total force of 8kN, this is given by the UDL multiplied by the distance that it acts over, $2\text{kN/m} \times 4\text{m} = 8\text{kN}$. Assuming that the upwards direction is a positive shear force. As the UDL is acting upwards this leads to a positive 8kN.
- iii. In diagram 1 there is shown a downwards force $P_1 = 10\text{kN}$ at a distance of 4m from the free end. This takes the 8kN vertically down to -2kN, $8\text{kN} - 10\text{kN} = -2\text{kN}$. The line is

vertically down as the force is acting downwards and is a point load so does not act over any distance.

- iv. The point moment at 5m does not affect the shear force diagram. However it does have an effect upon the magnitude of the support forces.
- v. At 6m where the wing is attached to the fuselage there is a reaction force of +2kN as the shear force diagram must return back to 0kN as the beam is in equilibrium.

2.1.2 Bending Moment Diagram



Diagram 3- Drawn Bending Moment Diagram

- i. The area under the shear force diagram gives the bending moment diagram. The UDL creates a positive slope curve that increases from 0kNm to 16kNm. This is due to the integration of the positive gradient straight line in the shear force diagram, it can also be shown by calculating the area of the triangle in the shear force diagram between 0-4m.
- ii. There is a gradual negative slope between 4m and 5m. As in the shear force diagram the area is negative as it is under the x-axis due to the point load.
- iii. At 5m there is a vertical drop of 10kNm due to the moment located at this point in diagram 1. The moment causes vertical drop as it a point moment acting over no distance. The drop is negative due to the moment being counter-clockwise.
- iv. Then from 5m to 6m there is a drop due to the negative area in the shear force diagram.
- v. At 6m the bending moment goes from 2kNm to 0 due to the reaction showing a reaction moment of 2kNm in a counter-clockwise direction.

2.2 MD Solids

In MDSolids go the 'Determinate Beam' option at the main menu. Then click on the cantilever beam that is restrained on the right (1st encircled option down in figure 5) . Enter the length of the beam as 6m. Apply the forces shown in diagram 1 using the options on the screen.

- 1) In figure 5 all of the options that are needed are circled in red.
- 2) Add the UDL in the upwards direction which is the option that is the 3rd circle down. Enter the start of load as 0 and the end of the load at 4 and check that the units are in m. Enter the 'load magnitude' as 2000 and check the units are in Newtons per meters (N/m.)
- 3) Add the point load which is the 2nd circled option down. Add the 'Load location' at 4m and the magnitude of 10,000N
- 4) Add the moment which is the 4th encircled option down. Add the 'Moment Location' as 5m and the 'Moment magnitude' as 10,000 Nm.
- 5) In the other window the Load, Shear Force and Bending Moment diagrams have been generated:

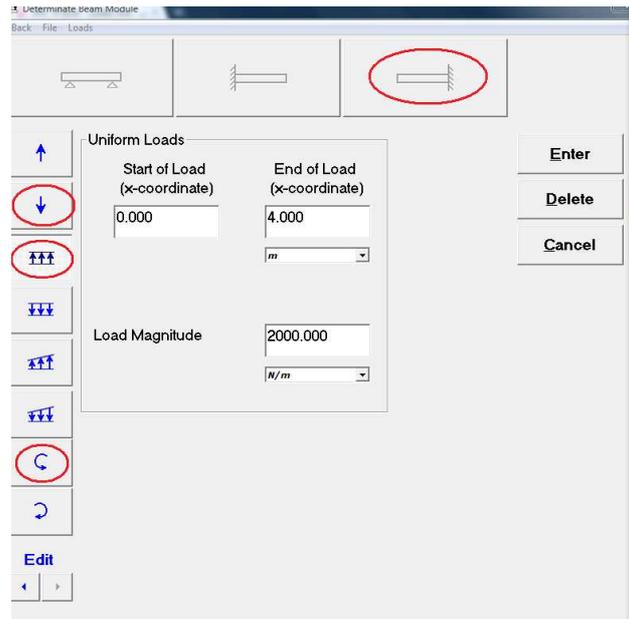


Figure 5 – Diagrams input loads menu

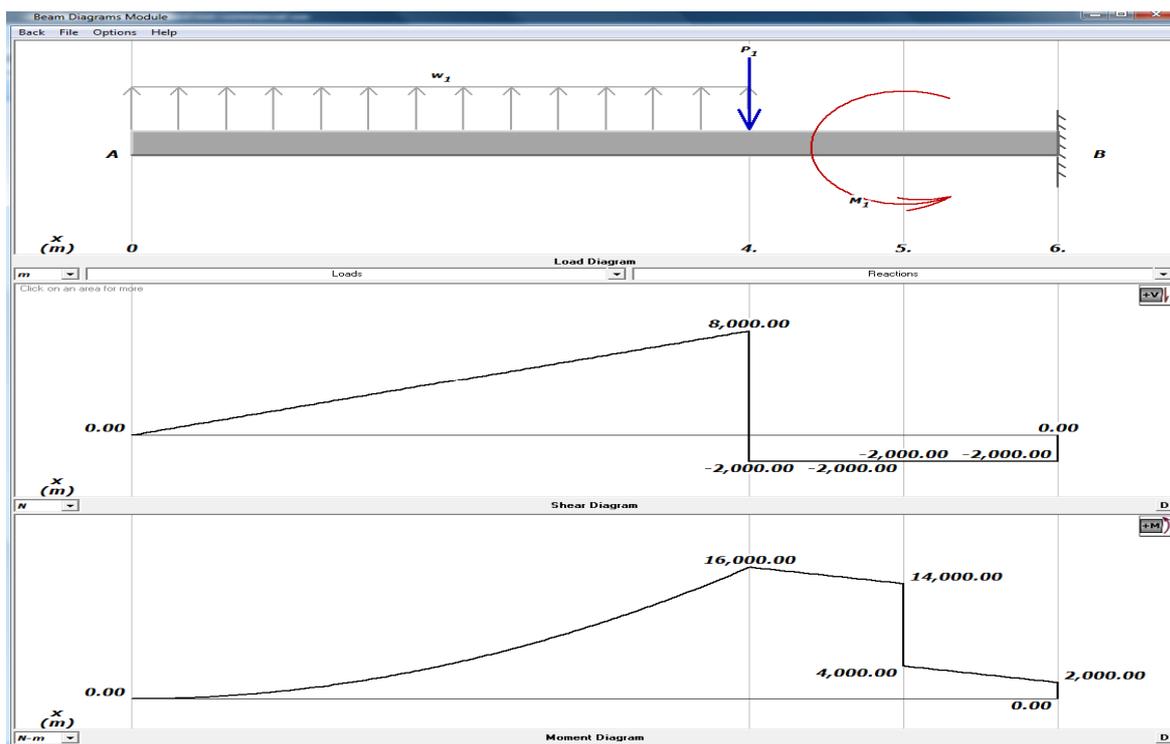
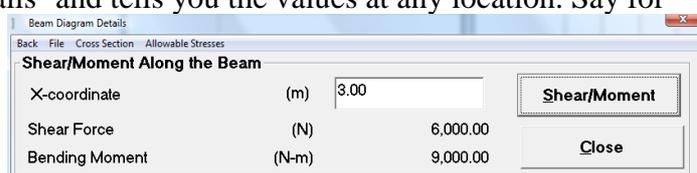


Figure 6 – MDSolids Shear Force and Bending Moment Diagrams

2.3 Comparison

The hand drawn diagrams and the MDSolids generated diagrams are the same. On MDSolids there is a tool called 'Beam Diagram Details' and tells you the values at any location. Say for an x-coordinate of 3m the had calculations show a shear force of 6kN



and a bending moment of 9kNm. The tool gives this answer – see figure 7.

Figure 7 – Beam diagram details

Question 3

Use the diagram shown in question 2 and calculate the shear force and bending moment at a distance of 4.5m from the free edge.

At this location calculate the principal stresses at Point A which is located on the vertical web at a distance of 20mm from the bottom edge of the flange.

Shear stress can be assumed to be constant over the cross section by dividing the shear stress by the CSA but in reality it is not.

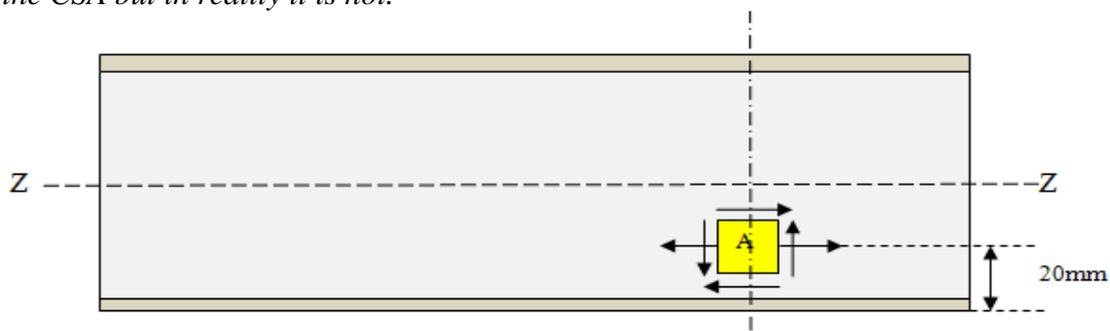


Figure 8 – Question 3

3.1 Shear force and Bending Moment at distance of 4.5m

Looking at diagram 2 the shear force at 4.5 m is -2kN.

Looking at diagram 3 the bending moment is 15kNm. As between 4m and 5m there is a change of 2000kNm and as it is a straight line the midpoint of the line at 4.5m must have a bending moment value of : $(16,000 - (2000/2)) = 15,000\text{Nm}$.

3.2 Simple Shear Stress

$\text{Shear stress} = \text{Shear force} / \text{CSA}$

From section 1.4 the CSA is $1,240\text{mm}^2$ but this needs to be converted into metres squared. Which is $1.24 \times 10^{-3} \text{m}^2$. From section 3.1 the shear stress is 2,000N. So:

$\text{Shear stress} = \tau = \frac{-2000}{1.24 \times 10^{-3}} = -1.613 \times 10^6 \text{ N/m}$ (it is negative as it is assuming the direction on the shear force on the right of the element is going down.

Shear stress is actually a lot more complicated than this as the shear flow is usually linear along the flanges and parabolic along the web.

3.3 Bending Stress

The distance point A is from the neural axis is 40mm.

From the bending moment diagram $M = 15,000\text{Nm}$

$$\sigma = \frac{My}{I_x}$$

$$\sigma = \frac{15,000 \times (40 \times 10^{-3})}{3.0903 \times 10^{-6}} = 194.116 \times 10^6 \text{ Nm} = 194.116 \text{ MPa}$$

3.4 Principal Stresses

Diagram 4 shows the stresses that are on Point A the Bending stress is the stress in the x-direction (σ_x) and the shear stress is represented by tau.

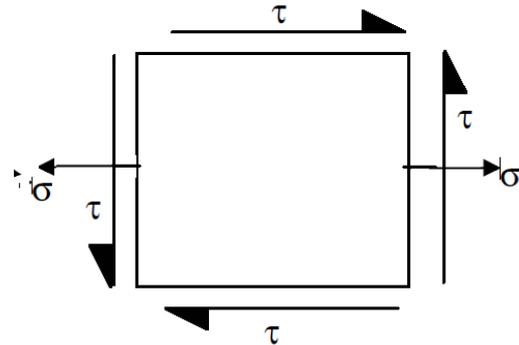


Diagram 4 – Principal Stresses

Where σ_1 is the maximum stress and σ_2 is the minimum stress. $\sigma_y = 0$ as there are no direct stresses in this direction. σ_x . The shear stress is now +ve as the direction is now in the opposite direction.

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$\sigma_1 = \frac{194.116 \times 10^6 - 0}{2} + \frac{1}{2} \sqrt{((194.116 \times 10^6 - 0))^2 + 4(1.613 \times 10^6)^2}$$

$$\sigma_1 = 194.124 \times 10^6 \text{ Pa}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$\sigma_2 = \frac{194.116 \times 10^6 - 0}{2} - \frac{1}{2} \sqrt{((194.116 \times 10^6 - 0))^2 + 4(1.613 \times 10^6)^2}$$

$$\sigma_2 = -13,300 \text{ Pa}$$

Using the following equation the maximum shear stress can be calculated:

$$\tau_{max} = \frac{1}{2} \sqrt{((\sigma_x - \sigma_y) + 4\tau^2)}$$

$$\tau_{max} = \frac{1}{2} \sqrt{((194.116 \times 10^6) + 4(1.613 \times 10^6)^2)}$$

$$\tau_{max} = 97.071 \times 10^6 \text{ Pa}$$

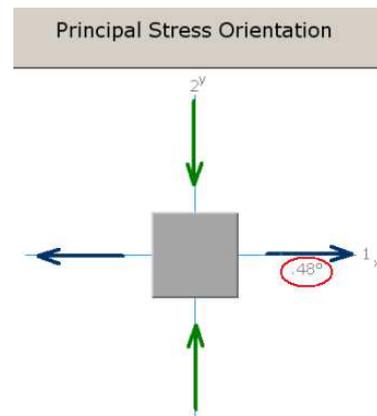
3.5 Angle of Principal Stresses

The equation of the angle in which the maximum and minimum stresses act is:

$$\tan 2\theta = \frac{2\tau}{(\sigma_x - \sigma_y)}$$

Re-arranging for theta:

$$\theta = \frac{1}{2} \left(\tan^{-1} \left(\frac{2\tau}{(\sigma_x - \sigma_y)} \right) \right)$$

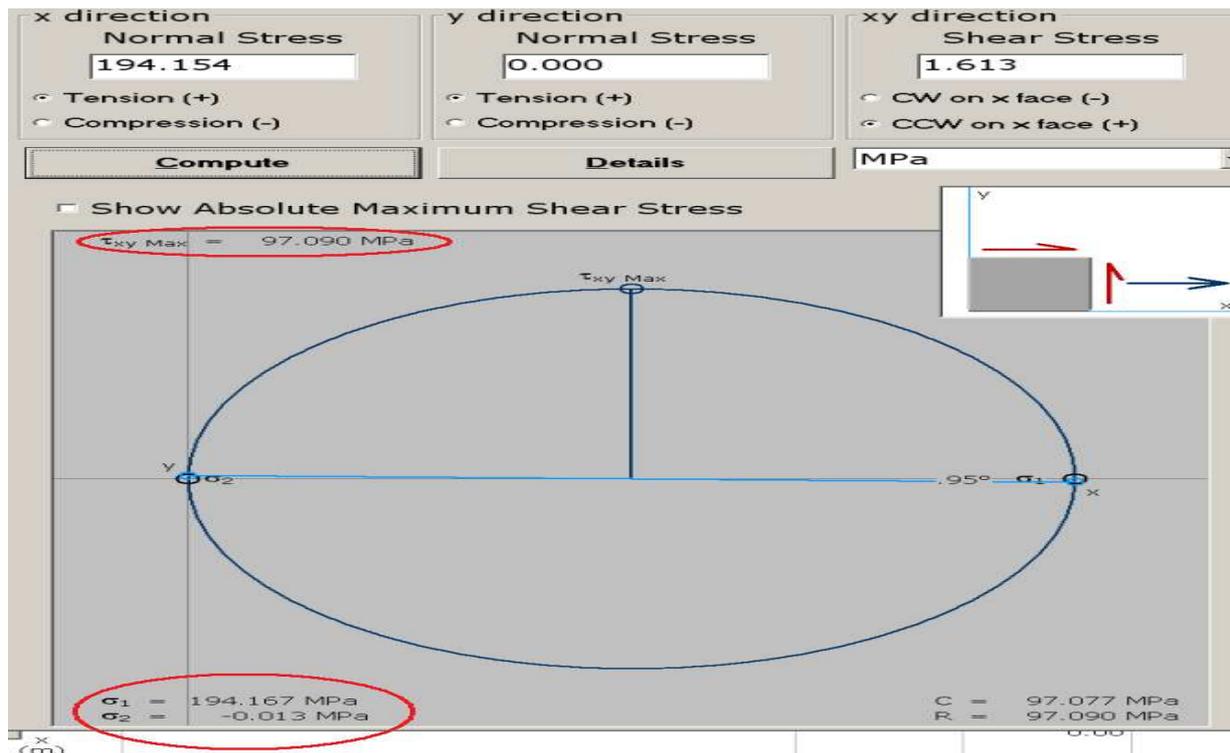


$$\theta = \frac{1}{2} \left(\tan^{-1} \left(\frac{2(1.613 \times 10^6)}{(194.116 \times 10^6)} \right) \right) = 0.47605^\circ$$

The angle for the other principal stress will be at 90 degrees to this so need to use this formulae:

$$\theta = \frac{1}{2} \left(\tan^{-1} \left(\frac{2\tau}{(\sigma_x - \sigma_y)} \right) + 180^\circ \right) = 90.476^\circ$$

3.6 MD Solids Principal Stresses



The principal stresses and the principal shear stress given by MDSolids and my hand calculations are very close. There may have been a small error due to the number of decimal places I took during the hand calculations.

Question 4

If a 60 degree ERSG rosette is located at Point A and one arm is in the z-direction, what would be the reading of the other gauges? What would be the principal strains at Point A.

4.1 Reading on the Gauges

4.1.1 Strains in X and Y Directions

Knowing the stress in the x-direction in 194.116MPa, the stress in the y-direction is 0 and that the young modulus is 70GPa and the poisons ratio is 0.3. We can use these equation to find the strain in the x and y direction.

$$\varepsilon_x = \frac{1}{E} (\sigma_x - \nu\sigma_y)$$

$$\varepsilon_y = \frac{1}{E} (\sigma_y - \nu\sigma_x)$$

$$\varepsilon_x = \frac{1}{70 \times 10^9} (194.116 \times 10^6 - 0) = \mathbf{2773 \mu\varepsilon}$$

$$\varepsilon_y = \frac{1}{70 \times 10^9} (0 - (0.3)(194.116 \times 10^6)) = \mathbf{-832 \mu\varepsilon}$$

4.1.2 Shear Strain

$$\text{Modulus of Rigidity} = G = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$E = 2G(1 + \nu)$$

$$G = \frac{E}{2(1 + \nu)}$$

$$G = \frac{70 \times 10^9}{2(1.3)} = 26.92 \times 10^9 \text{ Pa}$$

$$\text{Shear strain} = \gamma_{xy} = \frac{\text{shear stress}}{G} = \frac{1.613 \times 10^6}{26.92 \times 10^9} = \mathbf{-59.91 \mu\varepsilon}$$

4.2 Strains on the Gauges

Knowing that the angles on the gauges are 0 , 60 and 120 degrees we can now find the maximum stresses in each direction using the equation:

$$\varepsilon_\theta = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

We can also find the maximum shear strain at that angle using the equation:

$$\gamma_{nt} = -(\varepsilon_x - \varepsilon_y) \sin 2(\theta) + \gamma_{xy} \cos 2(\theta)$$

4.2.1 At 0 Degrees

We know that the strain at 0 degrees is **2773** micro-strain as it has been calculated in part 4.1.1.

The shear strain is:

$$\gamma_{nt} = -(\epsilon_x - \epsilon_y) \sin 2\theta + \gamma_{xy} \cos 2(0) = 59.91 \times 10^{-6} \epsilon$$

As $\sin 0 = 0$ and $\cos 0 = 1$.

4.2.2 At 60 Degrees

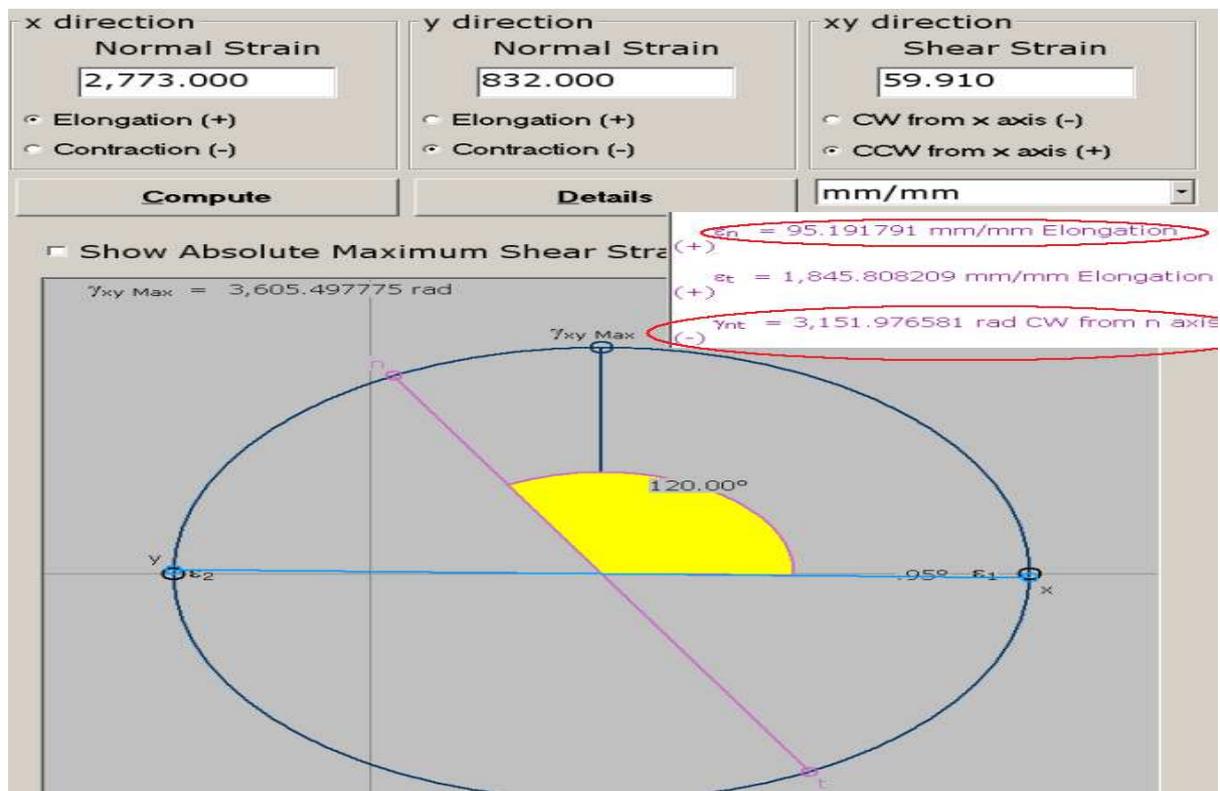
$$\epsilon_{60} = \frac{2773 \times 10^{-6} + (-832 \times 10^{-6})}{2} + \frac{2773 \times 10^{-6} - (-832 \times 10^{-6})}{2} \cos 2(60) + \frac{59.91 \times 10^{-6}}{2} \sin 2(60)$$

$$\epsilon_{60} = 970.5 \times 10^{-6} + (-901.5 \times 10^{-6}) + 25.9 \times 10^{-9} = 95.18 \times 10^{-6} \epsilon$$

Shear strain at 60 degrees:

$$\gamma_{nt} = -((2773 - (-832)) \times 10^{-6}) \sin 120 + 59.91 \times 10^{-6} \cos 120$$

$$\gamma_{nt} = -3151.95 \mu\epsilon$$



4.2.3 at 120 Degrees

$$\epsilon_{120} = \frac{2773 \times 10^{-6} + (-832 \times 10^{-6})}{2} + \frac{2773 \times 10^{-6} - (-832 \times 10^{-6})}{2} \cos 2(120) + \frac{59.91 \times 10^{-6}}{2} \sin 2(120)$$

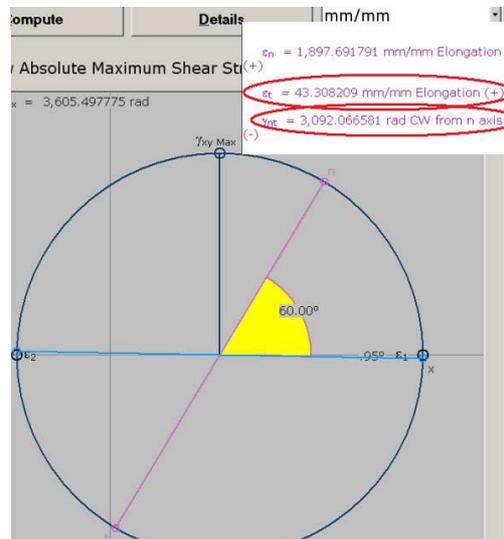
$$\epsilon_{120} = 970.5 \times 10^{-6} + (-901.5 \times 10^{-6}) + (-25.91 \times 10^{-6}) = \mathbf{43.1 \times 10^{-6} \epsilon}$$

Shear Strain at 120 degrees:

$$\gamma_{nt} = -\left((2773 - (-832)) \times 10^{-6} \right) \sin 240 + 59.91 \times 10^{-6} \cos 240$$

$$\gamma_{nt} = \mathbf{3084.27 \mu\epsilon}$$

The sign convention on MDSolids is wrong as you can only change it up to 90° so for the actual strain gauge n and t would be the opposite way round and the shear strain would be +ve.



4.3 Principal Strains

4.3.1 Maximum and Minimum principal strain

Using the equation:

$$\epsilon_{1 \text{ or } 2} = \frac{(\epsilon_x + \epsilon_y)}{2} \pm \sqrt{\left(\frac{(\epsilon_x - \epsilon_y)}{2} \right)^2 + \left(\frac{\gamma_{xy}}{2} \right)^2}$$

$$\epsilon_{1 \text{ or } 2} = \frac{(2773 \times 10^{-6} + (-832 \times 10^{-6}))}{2} \pm \sqrt{\left(\frac{(2773 \times 10^{-6} + (-832 \times 10^{-6}))}{2} \right)^2 + \left(\frac{59.91 \times 10^{-6}}{2} \right)^2}$$

$$\epsilon_1 = \mathbf{2773.25 \mu\epsilon}$$

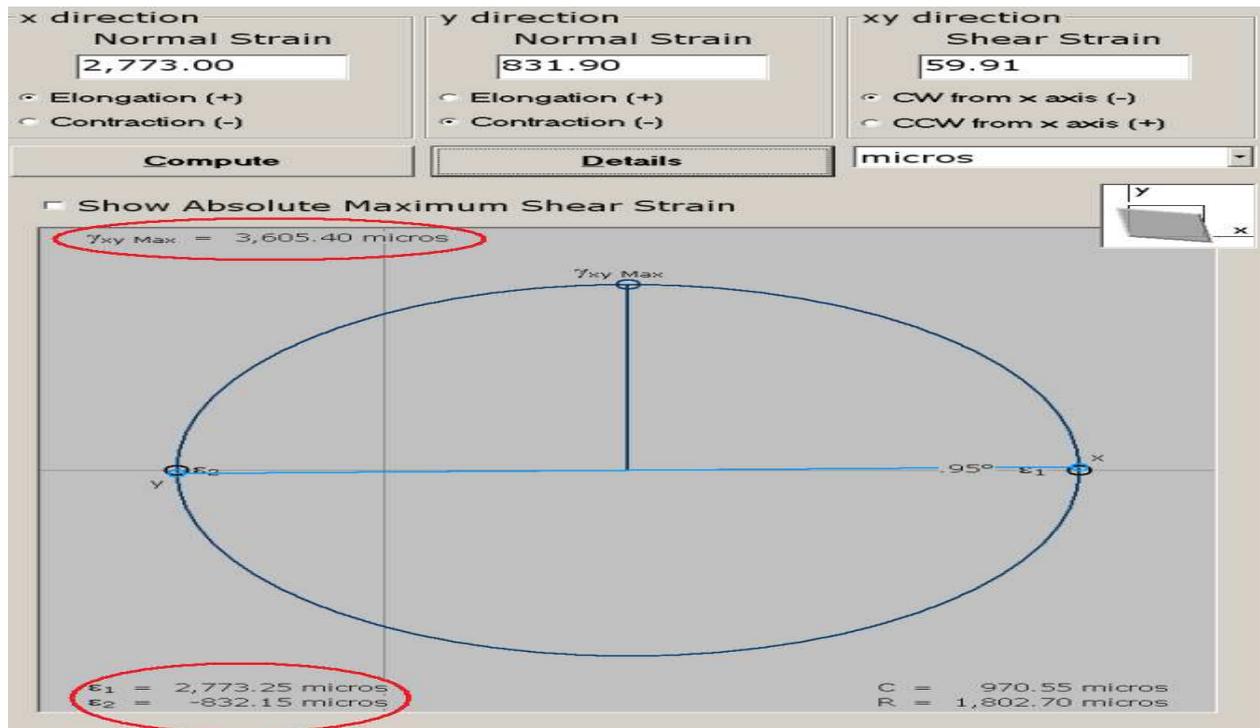
$$\epsilon_2 = \mathbf{-832.249 \mu\epsilon}$$

4.3.2 Maximum Shear Strain

Find the Principal shear strain by using this equation:

$$\frac{\gamma_{nt}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{nt} = 2\left(\sqrt{(1.0825 \times 10^{-3})^2 + (29.95 \times 10^{-6})^2}\right) = 3605.497 \mu\epsilon$$



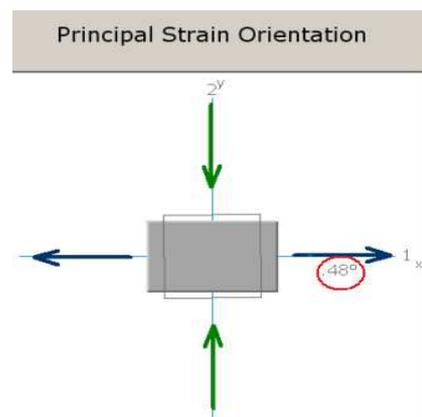
4.3.3 Shear Strain Orientation

$$\tan 2\phi = \frac{\tau_{xy}}{\epsilon_x - \epsilon_y}$$

$$\phi = \frac{1}{2} \left(\tan^{-1} \left(\frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} \right) \right)$$

$$\phi = \frac{1}{2} \left(\tan^{-1} \left(\frac{59.91}{3605} \right) \right) = 0.476^\circ$$

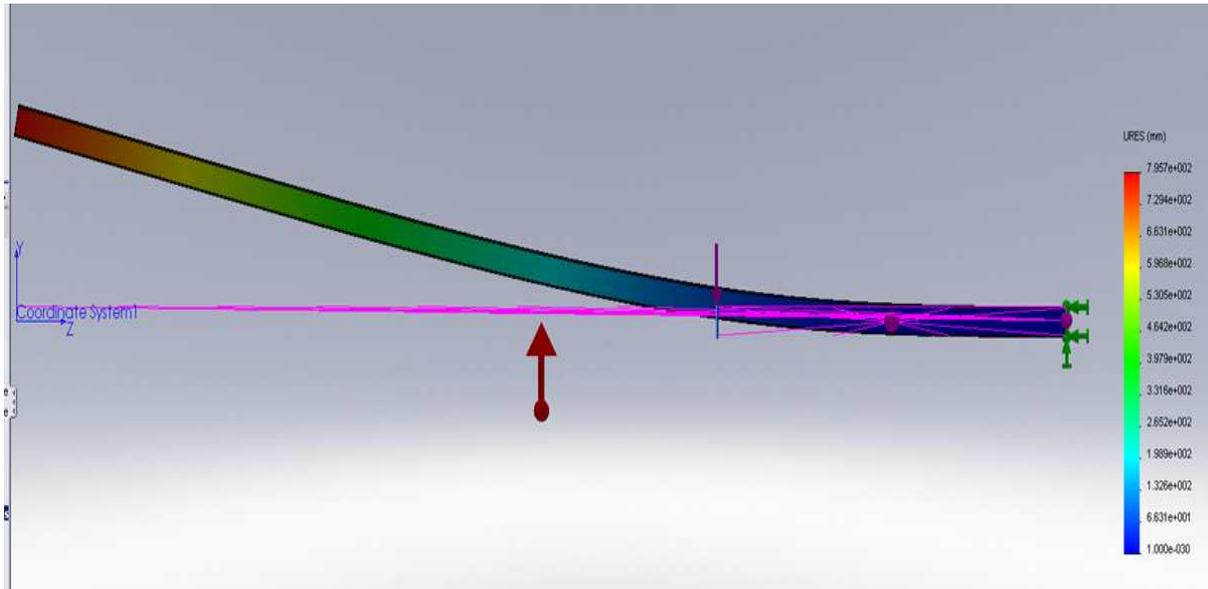
The other angle is at 90 degrees to this so at 90.476°



3D Finite Element Analysis

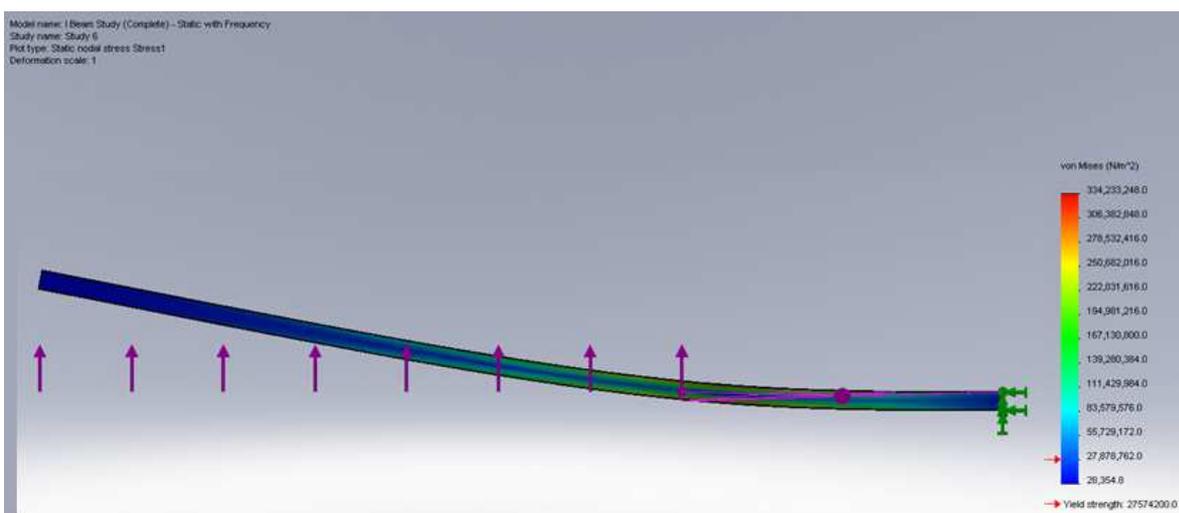
1.0 Static Loadings

1.1 Displacement



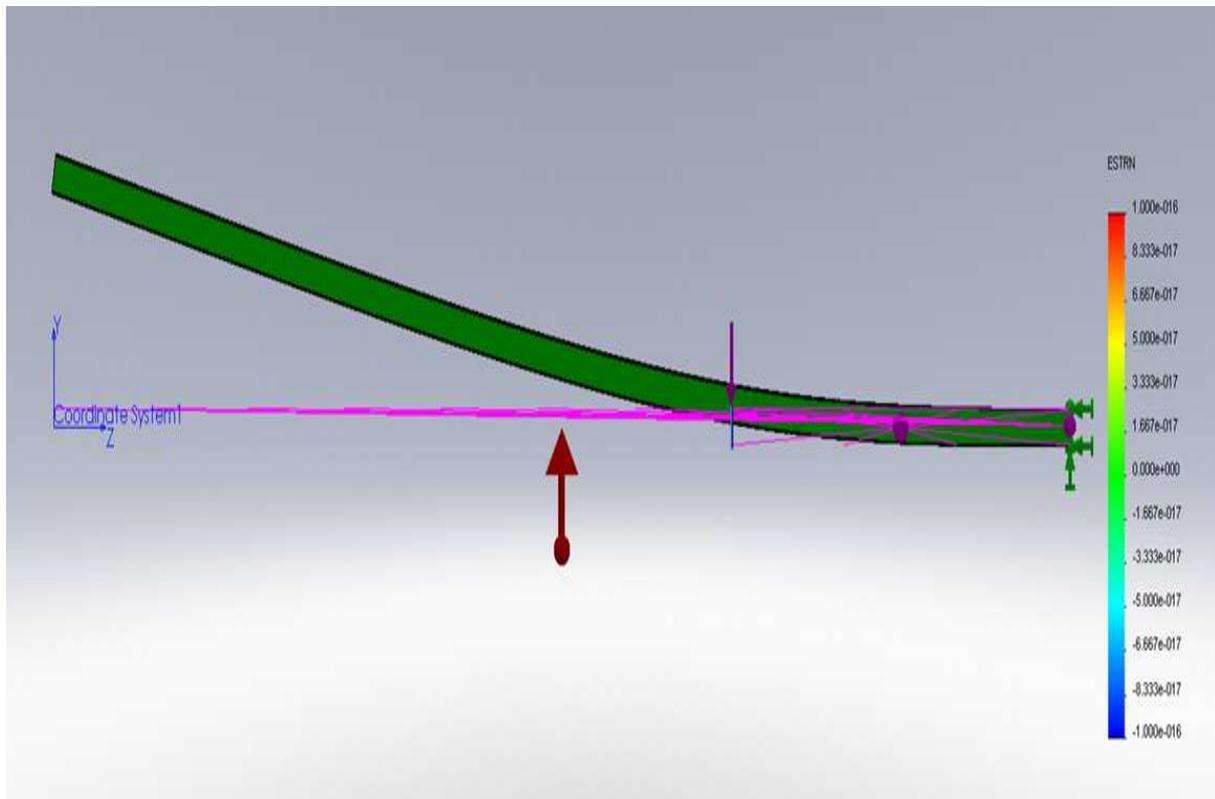
Here there is a maximum displacement of 795 mm at the end of the 6m. This displacement is quite large. This may cause the beam to fail due to the excessive bending causing tension and compression in at the top and bottom 'fibres' of the beam.

1.2 Stresses



This beam will fail as all the stresses are above the yield strength of the material. It will fail around the point where P1 is.

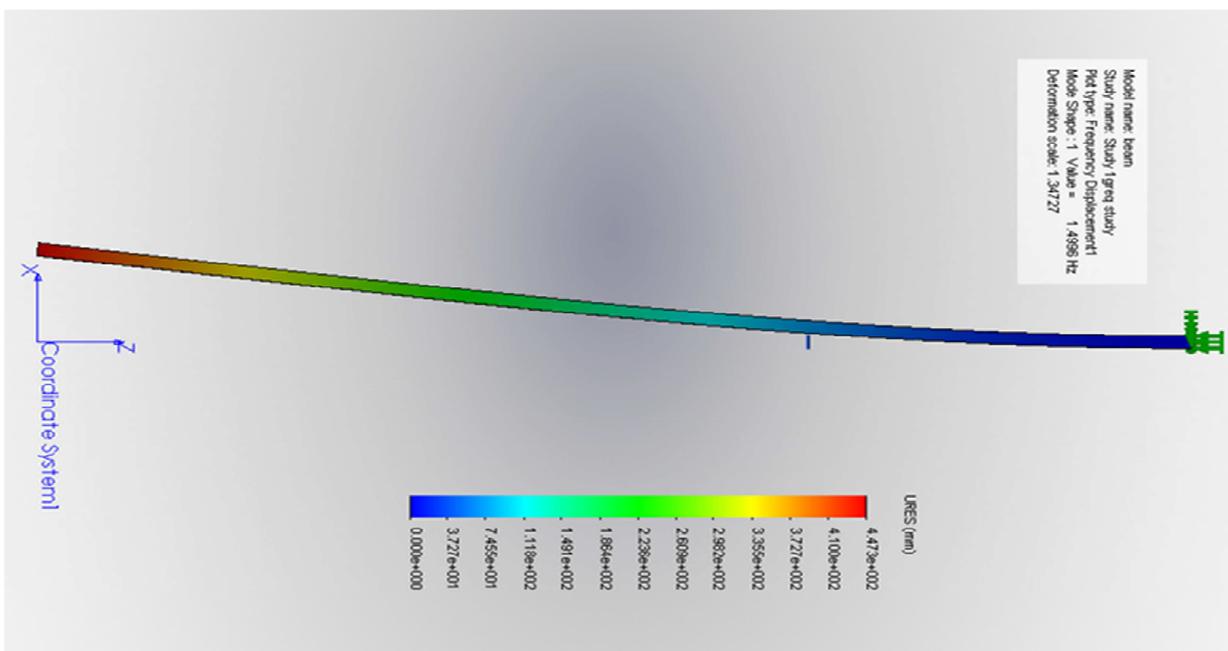
1.3 Strains



There appears to be very little strain this maybe due to the fact that the simulation has gone wrong due to the beam failing.

Natural Frequencies

1st Mode

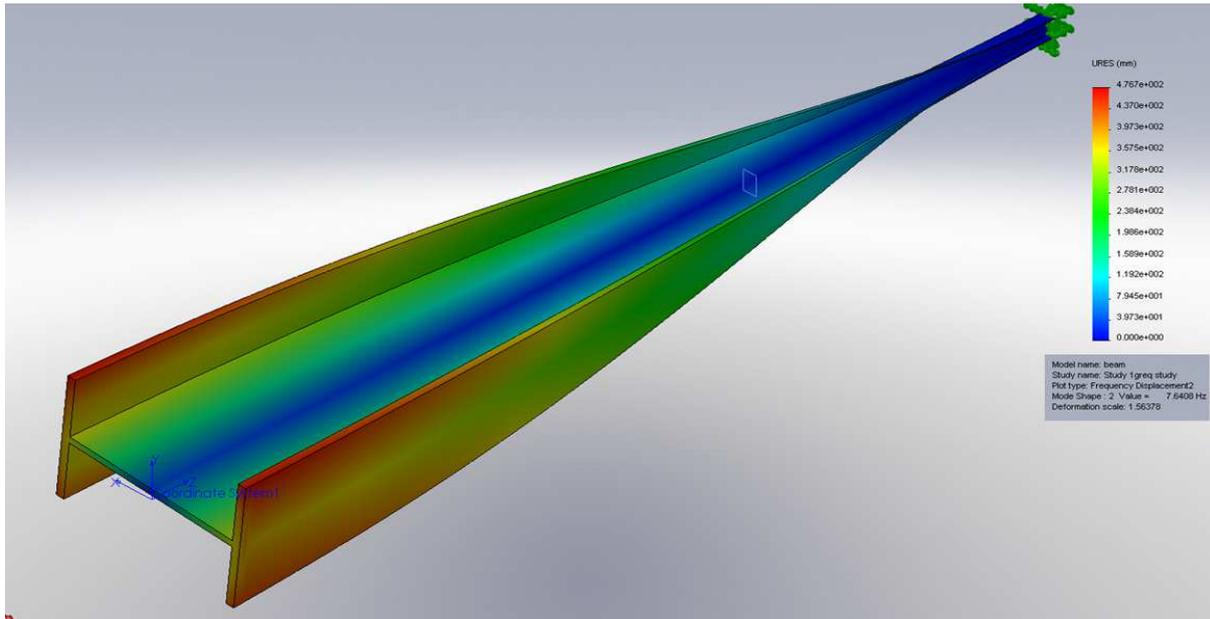


There is a displacement of 447mm in the x direction. This occurs at a frequency of 1.5Hz flutter in a wing occurs at about 2-3 Hz. So this beam may fail due to the natural frequency being the same as the excitation frequency, i.e. it gets to its resonant frequency. The stiffness of the beam may need to be changed or the mass can be changed. The stiffness of the beam is given by (the mass of the beam is 20kg assuming a density of 2700 kg/m³):

$$k = m (2\pi f)^2$$

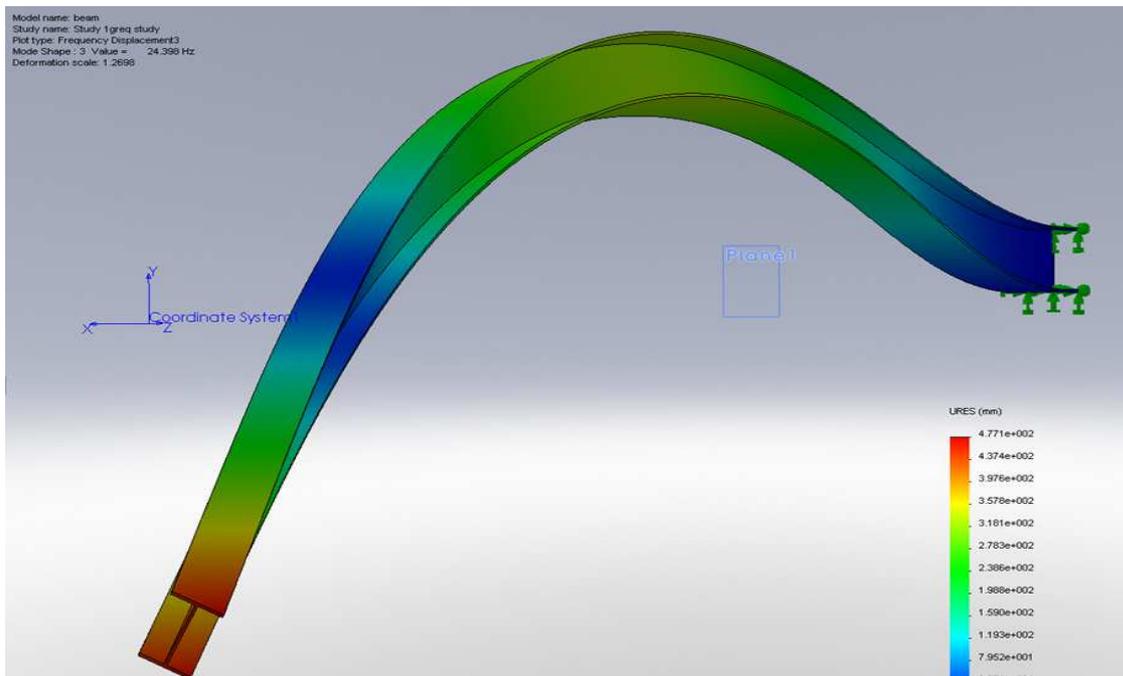
$$k = 20 (2\pi(1.5))^2 = 1760 \text{ N/m}$$

2nd Mode



The 2nd mode has a frequency of 7.64 Hz. This is above the frequency of excitation however there is about 90 degrees of twist in the beam. Also the web stretches, this tension will probably cause the beam to split down the web as there will also be large shear forces.

3rd Mode



This frequency is at 24.4Hz so excitation would probably never get to this frequency. Here there is deflection in all 3 dimension s x y and z and also twisting about the x and y axis.

Summary

The results that I have calculated by hand have been very close to those generated by MDSolids. The discrepancies may be due to me not using enough decimal places. I am not too sure about the solidworks simulation for the strains this is probably not correct. For the frequency I had to change the density of the Aluminium from about $2,700\text{kg/m}^3$ to around $110,000\text{ kg/m}^3$ this causes a UDL and does not take into account the locations of the loads so the frequencies maybe incorrect. Unfortunately I could not correctly figure out the shear stress at Point A using the method that uses shear flow. I used MDSolids and it gave an answer of 4.16MPa where as using the simple shear gave an answer of 1.613 MPa.

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